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Secondary instability of separated shear layers

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Abstract

The process through which a laminar flow undergoes transition to turbulence is of great fundamental and practical interest. Such a process is hugely complex as there are many diverse routes for a laminar flow to become turbulent flow. The transition process is usually initiated by flow instabilities - a primary instability stage followed by a secondary instability stage. This forms a rational framework for the early stage of a transition process and it is crucially important to understand the physics of instabilities leading to turbulence. This article reviews the results of studies on secondary instability of separated shear layers in separation bubbles and summaries the current status of our understanding in this area.

Keywords: Transition; Secondary instability; Separated shear layers; Separation bubbles; Primary instability

Nomenclature and Abbreviation

2D	two dimensional
3D	three dimensional
C	chord (m)
D	flat plat thickness (m)
DNS	Direct Numerical Simulation
f	Frequency (Hz)
KH	Kelvin-Helmholtz
LES	Large Eddy Simulation
TS	Tollmien–Schlichting
t	time (s)
t*	normalized time
Uo	free stream axial velocity (m/s)
u_s	boundary layer edge velocity at separation (m/s)
x_R	separation bubble length (m)
x	co-ordinate in axial (streamwise) direction (m)
y	co-ordinate in vertical direction (m)
z	co-ordinate in spanwise direction

Greek letters

θ_s	momentum thickness at separation (m)
$\Omega*$	normalized spanwise vorticity

1. Introduction

Despite continuous efforts by engineers, physicists and mathematicians for more than a century the problem of transition from laminar to turbulent flow is far from completely understood. Transitional studies are motivated by a need to understand this process and to apply this knowledge to the reduction and control of transition. For example, the low skin friction drag of laminar boundary layers is very attractive to the design engineers of high performance ground vehicles and aeroplanes. On the other hand, there are many cases where the enhanced mixing and heat transfer of turbulent flows are desirable such as in the combustion process.

Transition process is greatly influenced by flow configuration/geometry, by the presence of many different flow disturbances such as free-stream turbulence level, acoustic noise, pressure gradient, surface heating or cooling, suction or blowing of fluid from the wall and so on. Therefore transition process can take many possible routes.[1-14] Nevertheless under low flow environment disturbances it has been well established that transition process is initiated by flow instabilities via two stages:

i) Primary instability stage – small disturbances are

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amplified due to a linear instability called primary instability of the basic shear flow (Tollmien–Schlichting (TS) instability for attached boundary layers and Kelvin-Helmholtz (KH) instability for free shear layers) till they reach a size where nonlinear growth starts. This amplification can be in the form of exponential growth of eigenmodes, nonmodal growth of optimal disturbances, or nonmodal responses to forcing.

iii). Secondary instability – usually once a disturbance reaches a finite amplitude it often saturates and transforms the flow into a kind of new, possibly steady state. Very rarely the primary instability can lead the flow directly into a turbulent state and the new steady or quasi-steady flow becomes a base on which a new linear instability called secondary instability can occur. The growth rate of disturbances in the secondary stability stage is much larger than that of primary instability stage. It has been proposed that there is a possibility for the existence of a tertiary instability stage before breakdown to turbulence but there is little evidence so far to support this hypothesis.

This paper presents a review on studies of secondary instability of separated shear layers in separation bubbles (i.e. a shear layer as a result of a laminar boundary layer separation and forms a separation bubble, or a shear layer formed due to the separation of a laminar flow at a blunt/sharp leading edge leading to the formation of a separation bubble), trying to elucidate the instability mechanisms. Studies of secondary instability of free shear layers (mixing layers, shear layer of jets, and shear layers in wakes) will not be reviewed in this article but may be referred to during discussion. The author will try to review all significant works in this area but it is impossible to include every piece of work in this area and hence give my apology here to those whose work in this area is not included.

2. Secondary instability

2.1. A brief summary on secondary instability in attached boundary layers and free shear layers

Our understanding of the secondary instability stage in the transition process of separated shear layers in separation bubbles is relatively very limited compared with that in attached boundary layers and free shear layers. In two dimensional attached boundary layers, it has been well established that three dimensional disturbances become significantly amplified due to a secondary instability once two dimensional primary disturbances have reached sufficiently high amplitudes due to a primary instability (TS-wave), and the secondary instability was classified by Herbert[15] into three types as fundamental mode, subharmonic mode and detuned mode. The third type is rare and usually the secondary instability is either a fundamental (K-type instability) or subharmonic nature (H-type instability), i.e., the secondary instability amplifies disturbances with fundamental or subharmonic frequency with respect to the two dimensional one. This is also true for free shear layers and it has been demonstrated that fundamental type of secondary instability is associated with the formation of streamwise vortical structures called "ribs" while subharmonic type of secondary instability is associated with the merge of neighbour spanwise vortices called vortex pairing.[16-19] Most of the previous studies on the secondary instability of separated shear layers in separation bubbles have also shown the existence of both fundamental and subharmonic type instabilities.

2.2. Secondary instability of separated shear layers

Several studies on secondary instability of laminar separation bubbles formed on a flat plate where a laminar boundary layer develops over certain distance and separates due to an adverse pressure gradient have been carried out by Rist,[20] Rist and Maucher,[21] Rist et al.,[22] Maucher et al.,[23, 24] Marxen et al.,[25, 26] Rodriguez and Gennaro.[27] Both 2D and 3D disturbances were introduced in the studies on transition in separation bubbles on a flat plate[20-22] and it was observed that 3D disturbances at both fundamental and subharmonic frequencies with respect to the 2D TS-wave could be amplified due to a secondary instability. It was also shown that the 3D disturbances exhibit a reduced amplification after saturation of the 2D primary instability wave, which occurred for subharmonic as well as for fundamental 3D disturbance frequencies. Furthermore, they found that the secondary instability amplification rate in separation bubbles was difficult to detect since it was just about twice of the primary instability amplification rate whereas for attached boundary layers the secondary amplification is usually an order of magnitude larger than that of the primary amplification

In the numerical studies by Maucher *et al.*[23, 24] a 2D TS wave was forced by periodic wall normal suction and blowing in a disturbance strip upstream of the separation bubble. The amplitude of the TS wave grew rapidly in the separated region and eventually saturated in the reattachment region. They then superimposed 3D disturbances with a fixed spanwise wavenumber on the 2D TS-wave by short pulse like 3D excitation in the separation bubble with extremely low amplitude. A mechanism of secondary, temporal amplification of the 3D disturbances was observed. Similar to the secondary instability in attached boundary layers,[15] 3D amplification with subharmonic and fundamental frequency with respect to the forced 2D TS-wave could occur depending on the spanwise wavenumbers. The subharmonic mechanism was observed for low spanwise wavenumbers while the fundamental mechanism was found to dominate at high spanwise wavenumbers.

Jones *et al.*[28] studied numerically transition process of forced and unforced laminar separation bubbles on an aerofoil at incidence. They considered only fundamental type of secondary instabilities and identified that two possible secondary instabilities were active, one in elliptic regions of fluid flow (spanwise vortex core regions) and another in hyperbolic regions of fluid flow (regions between two consecutive spanwise vortices called Braid regions). It was observed in their study that those regions of pronounced disturbance growth appear to match regions of instability growth identified in mixing layers and bluff-body wakes. Therefore they argued that the mechanism of vortex core region instability is similar to that of elliptic instability[29] by which 3D disturbances are amplified (3D motions generated) in regions of 2D elliptical streamlines. The elliptic instability will result in a spanwise-periodic deformation of a vortex core and its mechanism is one of parametric resonance in which a normal mode, or pairs of normal modes, of oscillation on the undistorted circular flow becomes tuned to the underlying strain field. They also argued that the mechanism of braid region instability is similar to that of mode-B instability or called hyperbolic instability[30] which is a manifestation of the instability of 2D hyperbolic streamlines to 3D disturbances in the braid region. The hyperbolic instability leads to the formation of streamwise vortices periodically in the spanwise direction.

Maxren *et al.*[26] performed detailed analysis of vortex formation and its evolution in a forced laminar separation bubble induced by an adverse pressure gradient on a semi-infinite flat plate. They also demonstrated, similar to the study by Jones *et al.*,[28] that there were two secondary instabilities at work: the elliptic instability in the vortex core region leading to a spanwise deformation of the vortex core with a spanwise wavelength of the order of the size of the vortex; and another instability in the braid region which occurred for much higher spanwise wavenumbers compared to the elliptic instability, resulting in rapid disintegration of the spanwise vortex into small-scale turbulence. The suggested that this instability in the braid region is a kind of hyperbolic instability but also stressed that further study was needed to confirm this. In addition, they observed that both those secondary instability mechanisms occurred for fundamental and subharmonic frequencies with respect to the vortex shedding frequency. As mentioned in section 2.1 that in free shear layers subharmonic type of secondary instability is usually associated with the merge of neighbour spanwise vortices called vortex pairing but they pointed out that vortex pairing events did not occur in their experimental and numerical studies with three-dimensional perturbations.

A detailed experimental study on the nature of transition in a separation bubble on an aerofoil and manipulations of the resultant breakdown to turbulence through passive means of control was conducted by McAuliffe and Yaras.[31] The vortex pairing phenomenon initiated by a subharmonic type of secondary instability as mentioned above was clearly visualized as shown in Fig. 1 through a series of x–y plots of normalized spanwise vorticity. It can be seen from Fig. 1 that at $t^* = 0$, a vortex formed due to roll-up of the separated shear layer is observable and soon afterwards followed by a second vortex upstream at $t^* = 0.0544$. The subharmonic instability causes the second vortex to shift towards the higher-velocity side of the shear layer and as a result of this it travels downstream at a higher speed and catches up to the first vortex at $t^* = 0.2178$. It can be seen that those two vortices become stretched and elongated as they rotate about each other due to mutual induction of their vorticity fields and subsequently merge into a single vortex structure. The trajectories of the two vortex cores are also marked in Fig. 1, showing the difference in convection rates of the two vortices and the process

how they merge into one vortex. Furthermore during the merging process small scale turbulence is generated around the core of the new vortex as shown in Fig. 1 at t* = 0.2722.
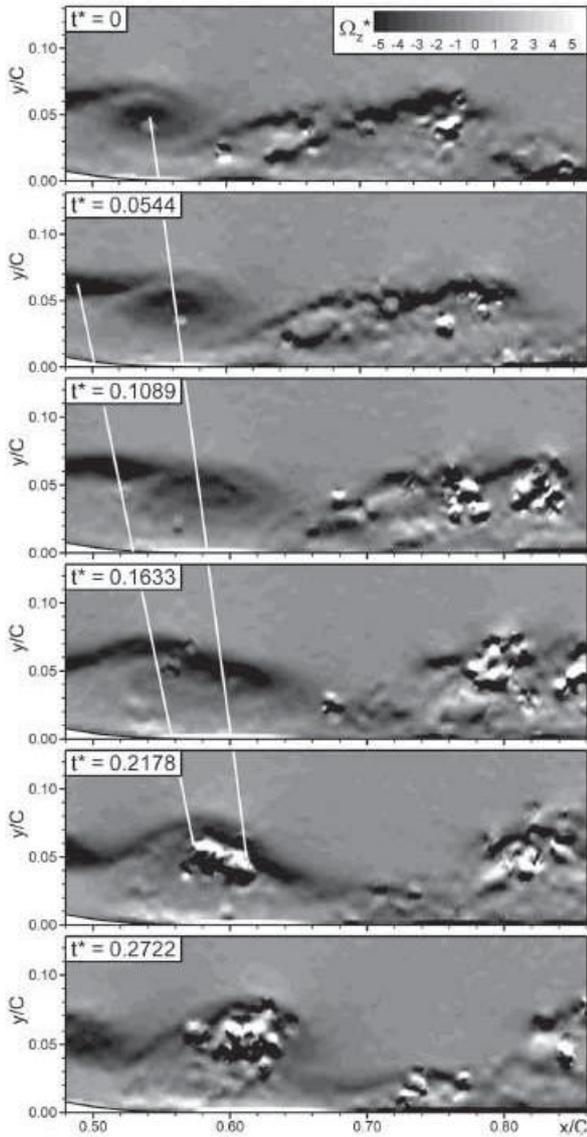

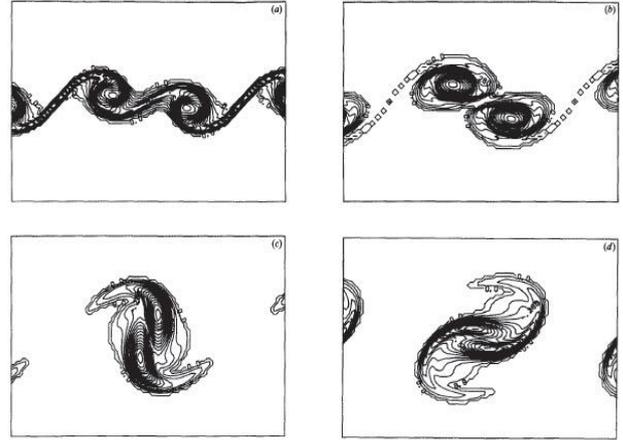**Fig. 1** Instantaneous spanwise vorticity showing the vortex pairing phenomenon in a laminar separation bubble.[31]

Fig. 2 Instantaneous spanwise vorticity contours showing the vortex pairing process: (a) t = 8, (b) t = 16, (c) t = 24, (d) t = 32.[17]

This kind of vortex pairing process is very well established in free shear layers and Fig. 2 shows a vortex paring process in a mixing layer study by Metcalfe *et al.*[17] Two distinct vortices can be clearly seen in Fig. 2a (t=8) and the upstream one travels faster than the downstream one, and get closer as shown in Fig. 2b (t=16). They merge into one vortex at t=24 as shown in Fig. 2c. Another secondary instability was also identified by Metcalfe *et al.*[17] which resulted in the bending of the core of the spanwise rolls (KH rolls), leading to the formation of the so called rib vortices. They demonstrated that those two secondary instabilities could coexist and compete against each other, and which secondary instability is at work or more dominant depends on flow history such as the initial disturbances, the relative amplitudes of each mode and so on.

Abdalla and Yang[32] performed LES of a laminar separation bubble on a flat plate with a blunt leading edge to investigate the primary and secondary instabilities of the transition process. They argued that a subharmonic secondary instability might be present as a vortex pairing process was just about identifiable in Fig. 3 showing that two vortices are about to merge (Fig. 3a) and after the merging (Fig. 3b). Nevertheless they also pointed out that this kind of pairing process was captured very rarely among the extensive data analysed and further study was needed to confirm this. In addition they also argued that possibly another secondary instability, similar to that identified by Metcalfe *et al.*,[17] was present as the so called rib vortices extending in the streamwise direction were observed as shown in Fig. 4.
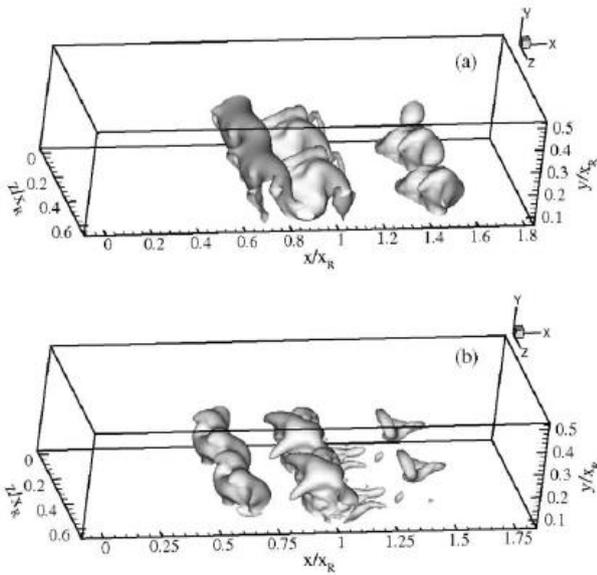
Fig. 3 Instantaneous pressure iso-surfaces showing the vortex pairing phenomenon in a laminar separation bubble: (a) t = 322.3D/Uo, (b) t = 390.2D/Uo.[32]

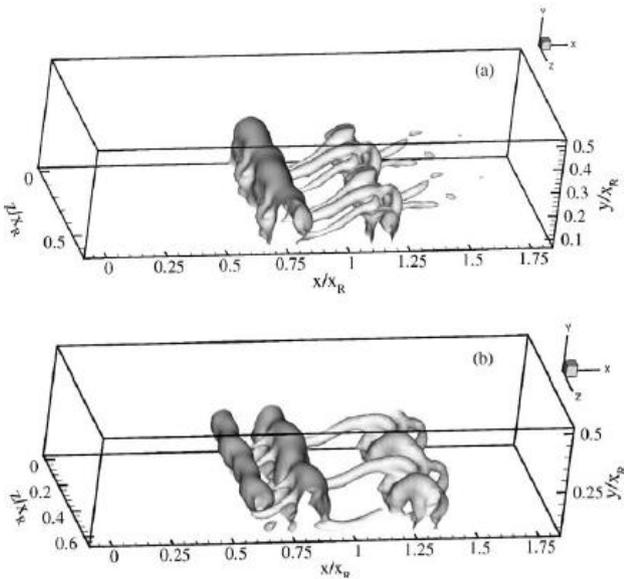**Fig. 4** Instantaneous pressure iso-surfaces showing the ribs vortices in a laminar separation bubble: (a) t = 305.4D/Uo, (b) t = 333.6D/Uo.[32]

Yarusevych *et al.*[33] carried out an experimental study of coherent structures in a separated shear layer over an aerofoil at low Reynolds numbers. Their results show that the separated shear layer rolls up due to the amplification of flow disturbances in the shear layer and

the roll-up vortices are shed subsequently at the frequency of the most amplified disturbances (the fundamental frequency). They found out that following the roll-up process, a subharmonic peak at half of the fundamental frequency was clearly observable in the velocity spectrum. Their further analysis indicated that this was due to the merging of the roll-up vortices, suggesting that a subharmonic type of secondary instability associated with the vortex pairing was at work.
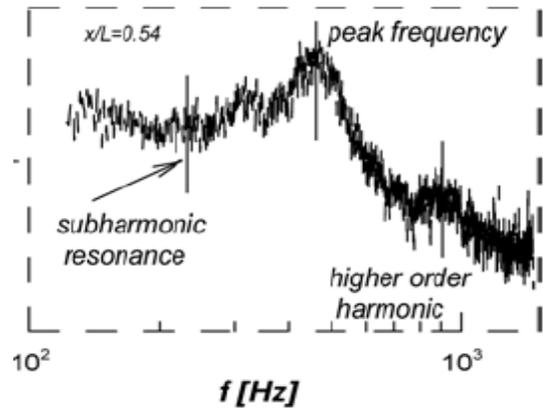

Fig. 5 Velocity spectrum showing three peaks associated with fundamental, subharmonic and high order harmonic.[34]

A laminar separation bubble formed on a flat plat due to an adverse gradient typical of ultra-high-lift turbine blade profiles was studied experimentally by Simoni *et al.*[34] They observed a distinct peak in the velocity spectra which was associated with the separated shear layer roll-up, matching the theoretical estimation for the KH instability frequency (fundamental frequency). In addition, a subharmonic peak at half of the fundamental frequency and also a higher order harmonic peak are just about visible in the velocity spectrum as shown in Fig. 5. They argued that the higher harmonic peak was due to the saturation process and the subharmonic peak was the result of the vortex pairing.

Serna and Lazaro[35] carried out an experimental study of the transition process of an unforced transitional separation bubble formed on a flat plate due to an adverse pressure gradient. It can be very clearly seen from the velocity spectra in Fig. 6 that a distinct frequency peak centred around a normalized frequency of 0.01 was present. This is associated with the shear layer inviscid instability (the most amplified disturbances at the fundamental frequency by the KH instability). Nevertheless they did not observe any

significant subharmonic development, which indicated that a subharmonic secondary instability associated with the pairing mechanism was not active. Instead, higher harmonics can be identified from Fig. 6 and they argued that those higher harmonics could be embedded in the primary instability mode and cannot a priori be interpreted as additional harmonic modes. They pointed out that the spectral data first showed harmonic growth followed by a rapid broadband energy transfer, suggesting the sudden formation of small-scale structure which might by due to the hyperbolic instability mechanism described in Marxen et al.[26]
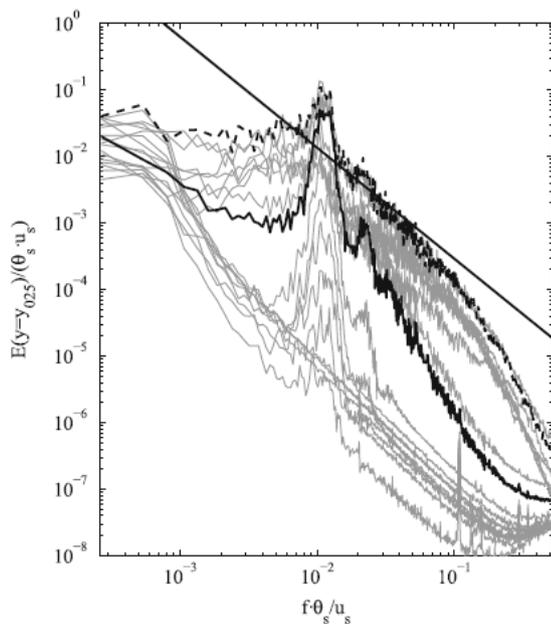

Fig. 6 Downstream evolution of the velocity spectra showing the fundamental frequency peak and the harmonic.[35]

Secondary instability of forced and unforced laminar separation bubbles was studied by Rodriguez and Gennaro.[27] Following on from their previous works[36, 37] they focused on the secondary instability of the 3D unforced laminar separation bubbles resulting from the amplification of the centrifugal instability by means of the temporal instability of 3D global modes. Without external forcing they found that laminar separation bubbles sustained an intrinsic 3D instability and their weakly non-linear analysis showed that it corresponded to a supercritical pitchfork bifurcation that broke the spanwise homogeneity of the base 2D bubbles. They performed secondary instability analyses of unforced bifurcated flow using two different approaches and the

same global oscillator was recovered independently by both approaches. However, it was shown that if a spanwise-averaged base flow was used in the secondary instability analysis then the global oscillator was not recovered. They also pointed out that only linear or weakly non-linear methods were used in their study and a fully non-linear study was needed to confirm their results.

He et al.[38] investigated 2D and 3D modal and non-modal instability mechanisms of massively separated flows around three NACA aerofoils at low Reynolds numbers based on global linear stability theory. They carried out secondary instability analysis of the laminar time-periodic nominally two-dimensional base flows using temporal Floquet theory, and the secondary instability occurred when the time-periodic flow became unstable to 3D perturbations. They identified two secondary instability mechanisms (two unstable modes) at quite different spanwise wavenumbers, a strong 3D secondary instability at large wavenumbers (short spanwise wavelengths, called SW instability) and a weaker one at small wavenumber (long spanwise wavelengths, called LW instability). Fig. 7 shows the amplitude functions of the spanwise component of the perturbation vorticity of the unstable three-dimensional SW and LW Floquet eigenmodes and Fig. 8 presents a perspective view of the 3D reconstruction of the flow field shown in Fig. 7.
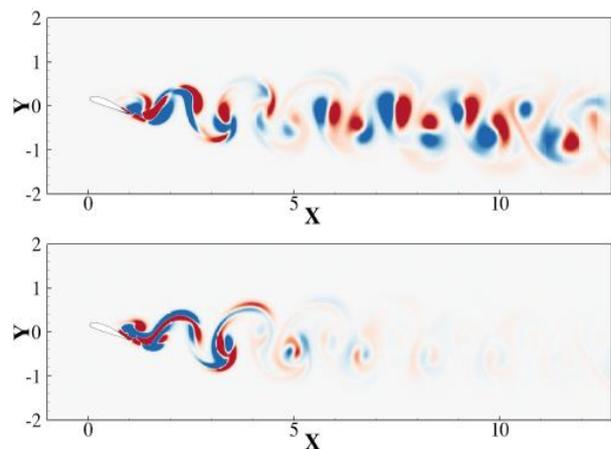

Fig. 7 Iso-surfaces of the amplitude functions of the spanwise component of the perturbation vorticity. Upper: Long-wavelength instability at spanwise wavenumber = 3. Lower: Short-wavelength instability at spanwise wavenumber = 11.[38]
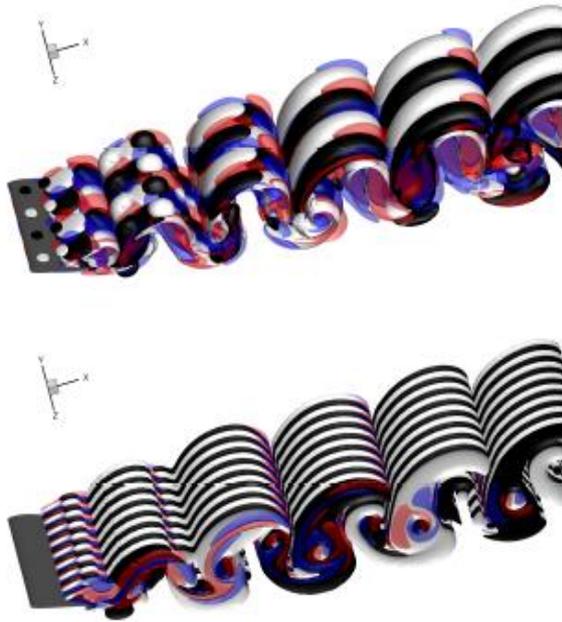
Fig. 8 3D reconstruction of the results of Fig. 7. Upper: Long-wavelength instability at spanwise wavenumber = 3. Lower: Short-wavelength instability at spanwise wavenumber = 11.[38]

3. Discussion

Transition is a hugely complex process and remains to be one of the unsolved problems in fluid dynamics. It may take many different routes with different mechanisms depending on flow configuration/geometry and background disturbances. It is worth pointing out that transition process initiated by flow instabilities via primary and secondary instability stages has been established based on the linear stability theory under very low flow disturbances (disturbance of very small amplitude to justify the linearization of the governing equations) while in many engineering flows transition occurs very rapidly and no instabilities may be observed (linear stability approach is not really applicable due to large amplitude flow disturbances). Furthermore the linear instability study has been traditionally approached by means of the modal scenario in which a linear problem is transformed into a eigenvalue problem and also the basic flow is assumed to be 1D (homogeneous in two of the three spatial directions, the so called parallel flow assumption, leading to the classical Orr-Sommerfeld equation). There have been significant developments in the past three decades and a different approach (formulation based on the linear initial-value problem), called nonmodal analysis, has emerged which can predict short-term disturbance growth (called transient growth or nonmodal growth).[39] The nonmodal approach has been used successfully to explain experimental observations that instabilities and transition scenarios happen over on substantially shorter timescale than that predicted by the traditional modal approach. In addition, the so called Global instability theory has been developed[40] since the 1D basic flow assumption is not valid in most engineering flow cases which are either 2D or 3D, more specifically BiGlobal instability analysis refers to (modal or nonmodal) analysis of 2D basic flow and TriGlobal refers to 3D. Although our understanding of transition process has been advanced and our knowledge of flow instability has been increased by those developments, plus weakly non-linear instability and numerical simulations (LES, DNS), there is still no breakthrough.

Extensive research has been carried out in the transition study of attached boundary layers and free shear layers over the past century while relative less research has been done in separated shear layers. Therefore transition processes in attached boundary layers and free shear layers are better understood compared with that in separated shear layers, and in fact instabilities proposed for separated shear layers have been mainly based on the results of free shear layer studies since the transition process in separated shear layers is in many ways very similar to that in free shear layers. Nevertheless, one needs to bear in mind that transition in separated shear layers with formation of separation bubbles is influenced by the presence of solid walls and depending on the cause of separation, e.g., when an attached boundary layer separates due to an adverse pressure gradient the transition process may have both viscous TS and inviscid KH characteristics whereas when laminar flows separate at a blunt leading edge the viscous TS instability will not be present.

It is more or less established that in separation bubbles, similar to what happens in free shear layers, the initial formed 2D or quasi-2D large spanwise vortices (KH rolls) are further destabilised by a secondary instability or the combination of several secondary instability mechanisms, result in the distortion of KH rolls in the spanwise direction and generation of 3D motions. So far the physical nature of the secondary instability

mechanisms of separated shear layers is not clear. The studies reviewed in the paper have suggested several possible secondary instabilities which could occur at both fundamental and subharmonic frequencies. One secondary instability mechanism identified initially in free shear layers, called the vortex pairing, was indeed observed in several studies on separated shear layers reviewed in this paper.[31-34] Several studies[20-25] reviewed here demonstrated that 3D disturbances at both fundamental and subharmonic frequencies with respect to the 2D TS-wave could be amplified due to a secondary instability but the associated instability mechanism was not fully understood. Two other studies[26, 28] reviewed in this paper suggested the existence of two secondary instabilities, the elliptical instability in the core region leading mainly to the deformation of vortex core and the hyperbolic instability in the braid region resulting in the generation of streamwise counter-rotating rib vortices with significant 3D motions. However, some studies in free shear layers did not support this idea as shown by Pierrehumbert and Widnall[41] that a fundamental type of secondary instability which they called translative instability, mainly localized in the core region, was responsible for both the deformation of the spanwise rolls and the generation of the counter-rotating streamwise rib vortices (a subharmonic secondary instability, corresponding to localized pairing of vortex tubes, was also identified in their study). Rogers and Moser[42] investigated the initial roll-up of a mixing layer and the development of three-dimensionality, and their results confirmed the findings by Pierrehumbert and Widnall[41] that the translative instability was responsible for the generation of continuous growth of both streamwise rib vortices and spanwise vortex deformation. They also pointed out the translative instability was not associated with an isolated region of the flow (braid or core region). Study by Nygaard and Glezer[43] indicated that the streamwise rib vortices were just a consequence of the bending spanwise vortices.

A very recent studies on instability mechanisms of massively separated flows around three NACA aerofoils at low Reynolds numbers based on global linear stability theory was carried out by He *et al.*[38] Two 3D secondary instability mechanisms (two unstable modes) were identified, a strong one at large wavenumbers (short spanwise wavelengths, called SW instability) and a weaker one at small wavenumber (long spanwise wavelengths, called LW instability). Nevertheless those two secondary instabilities occurred in the separated shear layers of aerofoil wakes which is not really within the scope of the current review paper. Considerable studies on secondary instability of wakes have been carried out and can be found elsewhere.[44-53]

It is evident that very limited research has been carried out to address specifically secondary instability of separated shear layers in separation bubbles despite significant amount studies on transition process in such flows. Most of those studies have not been directed at understanding the secondary instability mechanisms (mainly on primary instability, vortex shedding, coherent structures, breakdown etc.) and more research efforts are needed in this specific area, which can not only help to advance our fundamental understanding of the secondary instability mechanisms in separated shear layers but also may have significant impact on transition control.

4. Concluding remarks

Studies on secondary instability of separated shear layers in separation bubbles, i.e. a shear layer as a result of a laminar boundary layer separation, or a shear layer formed due to the separation of a laminar flow at a blunt/sharp leading edge, has been reviewed in this paper. A general discussion on instability studies based on the linear stability theory and the recent advances/developments in the linear stability theory has been given and our current understanding of the secondary instability mechanisms of separated shear layers is summarised.

Significant efforts have been made to understand the secondary instability in attached boundary layers and free shear layers while very limited efforts have been made towards understanding the secondary instability of separated shear layers in separation bubbles. Hence our current knowledge on the secondary instability mechanism is poor although a few possible secondary instabilities, a fundamental 3D type of instability, the vortex pairing mechanism at subharmonic frequency, elliptic and hyperbolic instabilities, have been proposed. There is no general consensus at all which instability or a combination of which instabilities are most likely at work, and a good understanding of their mechanisms is also lacking.

The transition process in separation bubbles are present and play a critical role in many different kind of practical flows. The primary instability stage is reasonably well understood and more focused research is needed to advance our fundamental understanding of the secondary instability mechanisms, through which we are a step closer to reveal the mystery of the transition process in such flows.

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