A Genetic Algorithm Approach to the Minimum Cost Design of Reinforced Concrete Flanged Beams Under Multiple Loading Conditions

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ABSTRACT

This paper presents results of the application of genetic algorithms to the minimum cost design of continuous beams cast in situ with reinforced concrete slabs to form an integral structure. A practical “problem-seeks-optimum design” approach requires full consideration of these rigidly jointed beam-and-slab connections, together with realistic multiple loading conditions and limit states as embodied in British and European Codes of Practice. The fitness function includes the cost of concrete, longitudinal and shear reinforcement, and the cost of formwork and labour. Results obtained so far have shown that genetic algorithms can be successfully applied to the minimum cost design of flanged beams, overcoming the difficulties associated with the discontinuity of the design equations and their complex inter-relationship with the design variables.

INTRODUCTION
Recent surveys (Cohn 1995), have shown a disappointing penetration of optimisation methods in practical structural design, especially in the field of optimum design of realistic 2D/3D reinforced concrete structural systems. One of main reasons for this, is that when compared with steel structures, the composite nature of reinforced concrete introduces another layer of complexity in the formulation of the optimum design problem. Furthermore, the complex interaction between the design variables and the discontinuous design equations makes these problems often impractical to be solved by applying traditional mathematical optimisation methods. Although these traditional methods have proven to be powerful optimisation tools, they have certain serious limitations (Fryer and Ceranic 1997), especially when applied to realistic structural systems with multiple loading conditions. In contrast, genetic algorithms have been shown to overcome these limitations with the advantage that stopping the algorithm short of reaching a real optimum still ensures a possible near-optimum solution. Furthermore, the “blindness” of genetic algorithms to the nature of applied structural problems, their ability to avoid derivates and linearisation errors, and their efficiency in dealing with discontinuous design equations makes them particularly attractive.

FORMULATION OF STRUCTURAL OPTIMISATION PROBLEM

Two possible section designs are considered assuming that each individual beam under the slab has either a ‘T’ or an inverted ‘L’ section consisting of a vertical web (or rib) surmounted by a flange as shown in Fig. 1. When the beams are resisting sagging moments, part of the slab acts as a compression flange and the members may be designed as T- or L- beams. With hogging moments the slab will be in tension and assumed to be cracked, therefore the beam must be designed as a rectangular section of width $b_w$ and overall depth $h$. In all cases, the
effective depth $d$ is given as the distance from the top of the compression zone to the centroid of the tension reinforcement.

![Figure 1. T-beam and L-beam](image)

When the slab does act as the flange its effective width $b_f$ is defined by empirical rules which are specified in British Standards (BS8110 1985) as follows:-

‘T’ beams - the lesser of the actual flange width, or the width of the web ($b_w$) plus $l_z/5$

‘L’ beams - the lesser of the actual flange width, or the width of the web ($b_w$) plus $l_z/10$

where $l_z$ is the distance taken between points of zero moments, and for a continuous beam may be taken as 0.7 times the effective span.

**Fitness Function**

The fitness function includes the cost of concrete, cost of steel and the cost of formwork together with their associated labour costs, mathematically presented as a function of the design variables $b_w$ and $h$. The total cost of the steel is the addition of the cost of the main reinforcement (determined using simplified rules for curtailment) and the cost of the shear reinforcement. Formwork and concrete costs correspond only to the material used in forming the web, as it is assumed that the costs pertaining to the flange are included in the construction of the surmounting slab. The proposed design procedure represents a practical
approach to estimating the total costs of the beam and includes the additional construction costs associated with making, fixing and striping formwork, steel fixing and material wastage. Taking account of all these costs the fitness function can be shown to be:–

\[ Z = Z_c + Z_s + Z_f \]  

(1)

where \( Z_c, Z_s \) and \( Z_f \) are the total cost of concreting, reinforcing and formworking respectively. Furthermore, the breakdown in the costs of concreting can be represented as:–

\[ Z_c = Z_{cm} + Z_{cw} + Z_{cl} \]

(2)

where \( Z_{cm} \) is the material cost, \( Z_{cw} \) is the cost allowance for wastage and \( Z_{cl} \) is the labour cost. Relating these individual costs to the design variables, the total costs of concreting can be shown to be:–

\[ Z_{cm} = \left[ C_c \left(1 + w_{fc}\right) + C_{cl}\right] b_w \left(h - h_f\right) \sum_{j=1}^{NS} L_j \]

(3)

where \( NS \) is the number of spans, \( C_c \) is the cost of concrete per unit volume, \( w_{fc} \) is the wastage allowance factor, \( b_w \) is the breadth of the web, \( h \) is the overall depth of the section, \( L_j \) is the effective length of the span and \( C_{cl} \) is the cost of labour per unit volume of concrete.

The cost of steel can be represented in similar manner as:–

\[ Z_s = Z_{sm} + Z_{sw} + Z_{sf} + Z_{sl} \]

(4)

where \( Z_{sm} \) is the material cost, \( Z_{sw} \) is the cost allowance for wastage, \( Z_{sf} \) is the steel fixing cost and \( Z_{sl} \) is the labour cost. Relating these individual costs to the design variables, the total costs of reinforcing can be shown to be:–

\[ Z_{sm} = \left[C_s \left(1 + w_{fs} + f_{fs}\right) + C_{sl}\right] \sum_{j=1}^{NS} \left(W_{j} + W_{s_j}\right) \]

(5)
where $C_s$ is the cost of steel per unit weight, $W_{lj}$ and $W_{sj}$ are the weights of longitudinal and shear reinforcement respectively, $w_f$ is the wastage allowance factor, $f_s$ is the steel fixing allowance factor and $C_l$ is the cost of labour per unit weight of steel.

Finally, the cost of formwork can be shown to be:

$$Z_f = Z_{tf} + Z_{tb} + Z_{wfp} + Z_{lm} + Z_{lfs}$$  \tag{6}$$

where $Z_{tf}$ and $Z_{tb}$ are the material cost of timber framing and boarding respectively, $Z_{wfp}$ is the cost allowance for wastage, fixing and props, $Z_{lm}$ is the cost of labour to make formwork and $Z_{lfs}$ is the cost of labour to fix and strip formwork. Relating these individual costs to the design variables, the total costs of formworking can be shown to be:

$$Z_{tf} = \left[T_f \left(C_{tf} + C_{tb}\right) \left(1 + w_f\right) \left(T_u + C_{lm}/T_u + C_{lfs}\right) \left[h_u + 2 \left(h - h_f\right)\right]\right]^{NS} \sum L_j$$  \tag{7}$$

where $C_{tf}$ is the cost of timber framing per unit volume, $C_{tb}$ is the cost of timber boarding per unit area, $T_f$ is the volume of timber framing per unit area of timber boarding, $T_u$ is the timber usage factor and $C_{lm}$ and $C_{lfs}$ are the labour costs to make and to fix and strip per unit area of timber respectively.

**Structural Analysis and Multiple Loading Conditions**

Continuous beams are analysed for the loading arrangements which give the maximum moments at the supports and within each span. These loading arrangement include maximum design load on all spans and maximum and minimum design loads on alternate spans. Using this analysis a design envelope is constructed showing at any point on the beam the worst effect that results from these loading arrangements. This design envelope is then used to determine the critical member forces required for the design process.

**Dimensional Constraints**
Lower- and upper-bound dimensional constraints are imposed on the design variables $b_w$ and $h$ to satisfy aesthetic requirements and practical design considerations. The lower-bound value for the overall depth of the beam is also set to ensure that the serviceability limit state deflection requirements will not be violated.

**Bending Reinforcement Design Equations**

For each span, the weight of the longitudinal steel $W_l$ is determined from the area of tension reinforcement $A_s$ and its corresponding length defined by the simplified rules of curtailment. The nature of the bending stress in the flange and the corresponding position of the neutral axis within the section gives three distinctive design cases for calculating $A_s$. In the first case, when the section is resisting sagging moments and the neutral axis falls within the flange, the reinforcement ratio $\rho_s$ is given by Eq. (8).

$$\rho_s = \frac{A_s}{b_f d} = \frac{M}{0.87 f_y z b_f d} \frac{1}{d}$$ \hspace{1cm} (8)

where $M$ is the bending moment due to ultimate loads, $f_y$ is the characteristic strength of reinforcement, $z$ is the lever arm, and $d$ is the effective depth of the section.

When the neutral axis falls below flange whilst resisting sagging moments, the shape of the compression zone is that of a $T$- or $L$- section and the design equation for calculating the reinforcement ratio is given by Eq. (9).

$$\rho_s = \frac{A_s}{b_w d} = \frac{M + 0.1 f_{cu} b_w d (0.45d - h_f)}{0.87 f_y (d - 0.5h_f)} \frac{1}{b_w d}$$ \hspace{1cm} (9)

where $f_{cu}$ is the characteristic concrete cube material strength.
Finally, when the section is resisting hogging moments the slab is in tension and the design equation is then given by Eq. (10).

\[
\rho_s = \frac{A_s}{b_w d} = \frac{M}{0.87 f_y z b_w d} \cdot \frac{1}{b_w d}
\]  
(10)

**Shear Reinforcement Design Equations**

To prevent punching shear type failure in the section the average shear stress \( \nu \) must not exceed the minimum value given by Eq. (11).

\[
\nu = \frac{V}{b_w d} \leq \min\left(0.8 \sqrt{f_{cu}}, 5 \text{ N/mm}^2\right)
\]  
(11)

where \( V \) is the shear force at the support due to ultimate loads.

For each span, the weight of shear reinforcement \( W_s \) is determined from the cross-sectional area of the links \( A_{sv} \) and their total length. Shear reinforcement in the form of nominal vertical links should be provided for the whole length of the beam according to Eq. (12).

\[
\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87 f_{yy}}
\]  
(12)

where \( s_v \) is the spacing of the links, and \( f_{yy} \) is the characteristic strength of link reinforcement.

For those parts of the beam where the average stress \( \nu \) exceeds \( (\nu_c + 0.4) \text{ N/mm}^2 \), where \( \nu_c \) is the ultimate shear resistance of concrete, the design links are provided according to Eq. (13).

\[
\frac{A_{sv}}{s_v} = \frac{b_w (\nu - \nu_c)}{0.87 f_{yy}}
\]  
(13)

**GENETIC ALGORITHM**
The optimisation algorithm systematically modifies tentative solutions of a design problem, producing new generations with a higher proportion content of the characteristics possessed by the fittest members of the previous generation. Two selection strategies are evaluated; the standard evolution approach and the elitist model. In the elitist model of selection, the best \( n \) members from the previous generation are preserved replacing the worst \( n \) individuals in the next generation. Each population run is modified using three standard operators; reproduction, crossover and mutation. Reproduction is based on the ranking principle with a choice of direct (deterministic) selection or remainder stochastic selection without replacement (Goldberg 1989). For the latter method, the integer part of the expected number of individuals is assigned directly, with additional copies being allocated using the remainder as probability selection criteria. For the one- or two-point crossover, the crossover site(s) for both parental strings are randomly chosen, and the alleles are then swapped between strings forming two new offsprings for the next generation. For uniform crossover a random mask is produced and information is exchanged with the parental string according to the position and percentage of zero’s in the mask to form a new offspring. Mutation operators are applied to the new string with a specified mutation probability per population and per gene size. The standard random mutation operator performs random alteration of the allele’s value, while the random mutation hill climb method repeats this process a specified number of times retaining only the beneficial mutations. The method of random mutation with directional hill climb was also incorporated, further exploring the benefits of random mutation in a positive direction. If the fitness improves, the vector difference between the old and new string is calculated and added to the new string. This process is repeated as long as the fitness improves or the number of steps is achieved. When used sparingly with reproduction and crossover operators, mutation can be seen as a safeguard against premature loss of important genetic material at a
particular position. This loss could lead towards a prematurely converged population and ‘local’ optimum problem, where mutation often represents the only means of redirecting the genetic algorithm search near the ‘global’ optimum design space.

6.6.2 Population Selection

Ranking principle has been used to determine a number of copies an individual can expect to receive according to its fitness. As stated in Chapter 6.2, ranking scheme not only that gives the maximum to average fitness normalisation, but also ensures that the fitnesses of the intermediate values are regularly spread out. Therefore, an effect of the ‘supefit’ individuals is negligible and overcompression ceases to be a problem.

In the approach adopted in this research, the probability of selection of an individual $p_{si}$ is decided to be

$$p_{si} = \frac{r_i}{\sum r_i} \quad (6.59)$$

where $r_i$ is the rank of the $i$-th individual and $N$ is the population size.

Once the probability of selection is established, individuals are then selected by simulating the spinning of the suitably weighted roulette wheel $N$ times. Mathematically speaking, the number of the expected copies of an individual $E_{si}$ is given by

$$E_{si} = p_{si}N \quad (6.60)$$

Given that the rank of individual with best (minimum) fitness is taken to be $N$, and rank of the individual with worst (maximum) fitness is $1$, the underlying trend of ranking is linear with corresponding fitness function required to be minimised. For the linear ranking schemes introduced in literature, most common suggestion is that the best solution is usually allocated
a probability of the selection of $2/N$, whilst the worst solution probability is constrained to be a zero, as outlined by Baker (1985). For the chosen probability of selection $p_s$ presented by Eq. 6.59, it will be proven that this is indeed true.

Term $\sum_{i=1}^{N} r_i$ represents sum of the first $N$ integer numbers, hence

$$\sum_{i=1}^{N} r_i = 1 + 2 + \ldots + N = \frac{N(N + 1)}{2}$$

(6.61)

From Eq. 6.59, for the individual with worst fitness whose rank is 1, the probability of selection is

$$p_{si} = \frac{1}{\sum_{i=1}^{N} r_i} = \frac{1}{\frac{N(N + 1)}{2}} = \frac{2}{N(N + 1)}$$

(6.62)

The expected number of copies from Eq. 6.60 is then

$$E_{si} = p_{si} N = \frac{2}{N + 1}$$

(6.63)

For sufficiently large size of population, i.e. $N > 20$, this value will converge to zero, therefore implicating the ‘death’ of the most unfit individuals.

On the other hand, probability of the selection of individual with best fitness whose rank is $N$ will be

$$p_{si} = \frac{N}{\sum_{i=1}^{N} r_i} = \frac{N}{\frac{N(N + 1)}{2}} = \frac{2}{(N + 1)}$$

(6.64)

The expected number of copies is

$$E_{si} = p_{si} N = \frac{2N}{N + 1}$$

(6.65)

For sufficiently large size of population, i.e. $N > 20$, the term on the right hand side of equation will converge towards value of 2. Therefore, for the adopted ranking scheme, the
most fit individuals will be given an opportunity to \textit{duplicate} themselves into the mating pool of the next generation. As it can be seen from the Eq. 6.60, the expected number of copies $E_{si}$ will not be an integer value, since the probability of selection $p_{si}$ is in general a fractional number. Therefore, a different methods in deciding how to assign the number of copies are reported in literature, as surveyed by Goldberg (1989), two of which are implemented and investigated in this research. The first one, named \textit{direct selection} method simply rounds the number of the copies to the nearest integer, whilst the latter one, called \textit{stochastic remainder without replacement} method assigns the integer part of the expected number of individuals directly, with additional copies being allocated using the remainder as probability selection criteria. Furthermore, two selection strategies are implemented and evaluated; the standard evolution approach and the \textit{elitist} model. In the \textit{elitist} model of selection, the best $n$ individuals from the previous generation are chosen and preserved, replacing the worst $n$ individuals in the next generation.

\textbf{6.6.3 Crossover (Recombination)}

Selection procedures, of course, do not introduce any new genetic material in the population, they solely decide on the formation of a mating pool. The crossover is the one mainly responsible for the introduction of the new genetic material, allowing offsprings to share some features from both parents. Therefore, a particular care and detailed investigation of different crossover operators have been performed in this research. As shown in Fig. 6.16, three main methods are implemented and results compared; \textit{one-point}, \textit{two-point} and \textit{uniform} crossover, explained in the Chapter 6.2. Furthermore, a probability of the crossover per population is introduced, giving opportunity to some parental strings to pass whole of the genetic material to the offspring by simple duplication. The facilities to change nature and
probability of the crossover at any point of program run are developed, proven to be of a significant impact in the stage of comparison and testing of results.

6.6.4 Mutation

Once crossovered, a mutation operators are applied to the new string according to the specified mutation probability per population and per gene size. Types and probabilities of mutation are assigned from the mutation control form, as shown in Fig 6.17.

The standard random mutation operator performs random alteration of the allele’s value, while the random mutation hill climb method repeats this process a specified number of times retaining only the beneficial mutations. The method of random mutation with directional hill climb was also incorporated, further exploring the benefits of random mutation in a positive direction. If the fitness improves, the vector difference between the old and new string is calculated and added to the new string. This process is repeated as long as the fitness improves or the number of steps is achieved. When used sparingly with reproduction and crossover operators, mutation can be seen as a safeguard against premature loss of important genetic material at a particular position. This loss could lead towards a prematurely converged population and ‘local’ optimum problem, where mutation often represents the only means of redirecting the genetic algorithm search near the ‘global’ optimum design space.

**NUMERICAL EXAMPLE**

Fig. 2 shows a three-span continuous T-beam subjected to three loading combinations. The length of each span and the corresponding loads (excluding self weight) are indicated in the figure. The lower- and upper-bounds of breadth of web and overall depth are given as 250
and 500 mm, and 500 mm and 900 mm respectively. Actual flange width is 4000 mm, thickness of flange is 200 mm and cover to reinforcement is 40 mm. The partial safety factors for imposed and dead load are 1.6 and 1.4 respectively with a minimum partial safety factor of 1.0. Characteristic concrete cube material strength \( f_{cu} \) is 30 N/mm\(^2\), \( f_y \) is 460 N/mm\(^2\) and \( f_{vy} \) is 250 N/mm\(^2\).

![Three-Span Continuous T-Beam](image)

Fig. 2 Three-Span Continuous T-Beam

The costs associated with concreting, reinforcing and formworking are presented in Table 1.

<table>
<thead>
<tr>
<th>Concreting</th>
<th>Rate</th>
<th>Reinforcing</th>
<th>Rate</th>
<th>Formworking</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of concrete (£/m(^3))</td>
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<td>Cut, bent &amp; bundled (£/tonne)</td>
<td>275</td>
<td>Cost of timber framing (£/m(^3))</td>
<td>285</td>
</tr>
<tr>
<td>Wastage (%)</td>
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<td>Wastage (%)</td>
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<td>Timber framing (m(^3)/m(^2))</td>
<td>0.055</td>
</tr>
<tr>
<td>Labour (£/m(^3))</td>
<td>36</td>
<td>Fixing Accessories (%)</td>
<td>5</td>
<td>Cost of timber boarding (£/m(^2))</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Labour (£/m(^3))</td>
<td>245</td>
<td>Wastage + fixings + props (%)</td>
<td>15</td>
</tr>
</tbody>
</table>

**Table 1. Structure of Costs for Three-Span Continuous Beam**

The size of the population was fixed at 40 with the total number of generations for each run being limited to 90. The probability of crossover per population was set at 80%, with the where \( n \) is the number of genes in the population member, to ensure that on average at least one gene mutation occurs. Fig. 3 shows the convergence history of the minimum value of the \( n \).
fitness function when employing the standard evolution approach and elitist model. In the elitist model, the 5 fittest members were retained for the next generation. Both selection methods used two-point crossover and random mutation. The elitist model yields better convergence tendency (that is, higher convergence rate) than the standard evolution approach.
without replacement. In both cases, elitism (best 5 members), two-point crossover and random mutation were used. No one method demonstrates a better overall convergence tendency although in this case the direct selection method shows an initially higher convergence rate. Fig. 5 shows the convergence history of the minimum value of the fitness function for one-point, two-point, 20%-uniform and 60%-uniform crossover. In all cases, elitism (best 5 members) and random mutation were used. The two-point and 20%-uniform crossover yield better convergence tendency, although the 60%-uniform crossover initially shows a higher convergence rate than the two-point crossover. One-point crossover has the poorest convergence tendency. Fig. 6 shows the convergence history of the minimum value of the fitness function for random mutation, random mutation hill climb and directional hill climb. In all cases, elitism (best 5 members) and two-point crossover were used. The number of steps for both random and directional hill climb was set to 5. The random hill climb and directional hill climb yield better convergence tendency than the random mutation method. Furthermore, the directional hill climb method shows the highest rate of convergence.

CONCLUSIONS

The presented results illustrate the performance of genetic algorithm optimisation applied to the minimum cost design of reinforced concrete continuous $T$- and $L$-beams. The proposed programming problem is a highly practical approach to the design process, incorporating multiple loading conditions and material and labour costs associated with concreting, reinforcing and formworking. This design approach, which has been neglected by traditional structural optimisation, has potential for being developed into the process of automated optimum design. Indeed, the ability of genetic algorithms to avoid gradient computations and rapidly search the entire feasible region independent of the starting point, provides the
designer with a useful set of tools that can be used to find beam sections that are optimal in a practical sense. The results of numerous studies have shown that the performance of the evolutionary search can be enhanced by an appropriate choice of parameters for genetic algorithms. Good results have been obtained using the elitist model which has been shown to perform better that the standard evolution approach. However, great care needs to be taken in deciding how many members to retain to avoid the danger of premature forced convergence. Uniform crossover generally achieved the best convergence rate, whilst the random hill and directional hill climbing methods have shown advantages, mutating the genes in a beneficial manner that generally improves convergence. A suggestion for future work is to study the developing of algorithm for an integral structure, incorporating the design of the reinforced concrete slabs into the optimisation of these rigidly jointed structural connections.

REFERENCES

British Standards, (1985), Structural Use of Concrete, BS 8110 : Part 1